

# REVISITING PERFECT FLUID DARK MATTER: OBSERVATIONAL CONSTRAINTS FROM OUR GALAXY

Alexander A. Potapov<sup>1,a</sup>, Guzel M. Garipova<sup>1,b</sup> and Kamal K. Nandi<sup>1,2,3,c</sup>

<sup>1</sup>Department of Physics & Astronomy, Bashkir State University, Sterlitamak  
Campus, Sterlitamak 453103, RB, Russia

<sup>2</sup>Zel'dovich International Center for Astrophysics, M. Akmullah Bashkir State  
Pedagogical University, Ufa 450000, RB, Russia

<sup>3</sup>Department of Mathematics, North Bengal University, Siliguri 734 013, WB,  
India

<sup>a</sup>E-mail: potapovaa@mail.ru

<sup>b</sup>E-mail: goldberg144@gmail.com

<sup>c</sup>E-mail: kamalnandi1952@yahoo.co.in

PACS numbers: 04.20 Gz, 04.50 + h, 04.20 J

---

## Abstract

We revisit certain features of an assumed spherically symmetric perfect fluid dark matter halo in the light of the observed data of our galaxy, the Milky Way (MW). The idea is to apply the Faber-Visser approach of combined observations of rotation curves and lensing to a first post-Newtonian approximation to "measure" the equation of state  $\omega(r)$  of the perfect fluid galactic halo. However, for the model considered here, no constraints from lensing are used as it will be sufficient to consider only the rotation curve observations. The lensing mass together with other masses will be just computed using recent data. Since the halo has attractive gravity, we shall impose the constraint that  $\omega(r) \geq 0$  for  $r \leq R_{\text{MW}}$ , where  $R_{\text{MW}} \sim 200$  kpc is the adopted halo radius of our galaxy. The observed circular velocity  $\ell$  ( $= 2v_c^2/c_0^2$ ) from the flat rotation curve and a crucial adjustable parameter  $D$  appearing in the perfect fluid solution then yield different numerical ranges of  $\omega(r)$ . It is demonstrated that the computed observables such as the rotation curve mass, the lens mass, the post-Newtonian mass of our galaxy compare well with the recent mass data. We also calculate the Faber-Visser  $\chi$ -factor, which is a measure of pressure content in the dark matter. Our analysis indicates that a range  $0 \leq \omega(r) \leq 2.8 \times 10^{-7}$  for the perfect fluid dark matter can reasonably describe the attractive galactic halo. This is a strong constraint indicating a dust-like CDM halo ( $\omega \sim 0$ ) supported also by CMB constraints.

**Keywords:** *Dark matter, perfect fluid, equation of state, galactic masses*

# 1 Introduction

A few years ago, in Ref.[1], a perfect fluid dark matter model was developed that was shown to have many attractive theoretical aspects. The solution may be thought of as a dark matter induced spacetime embedded in a static cosmological Friedmann-Lemaître-Robertson-Walker (FLRW) background<sup>1</sup>. The motivation for developing an isotropic perfect fluid model (we leave open the question of particle identity of dark matter) came from the fact that predictions from such model at stellar and cosmic scales have been observationally well corroborated so far. More recently, Harko and Lobo [2] investigated dark matter as a mixture of two non-interacting perfect fluids, with different four-velocities and thermodynamic parameters. González-Morales and Nuñez [3] compared two different dark matter models, one is a perfect fluid and the other is a scalar field [3]. See also [4].

The model considered here assumes that a spherical dark matter distribution is the only gravitating source. This assumption is of course an oversimplification since, although the bulge is quite spherical and is dominated by old stars, the Milky Way has a strongly flattened stellar distribution. However, we know from the vertical velocity dispersion of stars as a function of distance from the disk plane that the local disk mass density is almost identical to the sum of the densities that can be attributed to stars, gas and stellar remnants. Therefore, there is practically little dark matter hidden in the disk. Hence, to explain the rotation curve measurements, we are forced to assume that dark matter resides in the halo region dominating its mass, *is* spherically distributed and, if it is non-baryonic, would not be expected to collapse into a disk-like structure.

Specifically, the hypothesis of dark matter arose because the Newtonian circular velocity  $v_c^2 = \frac{GM(r)}{r}$  of circularly moving probe particles caused by the luminous mass distribution  $M(r)$  is not supported by observations [5,6]. The circular velocity becomes nearly flat,  $v_c^2 \simeq \text{constant}$ , at distances far away from the center (halo region), which is possible only if  $M(r) \propto r$ . Therefore, almost every galaxy is assumed to host a large amount of non-luminous matter, the so called gravitational dark matter, consisting of unknown particles not included in the particle standard model, forming a halo around the galaxy. Naturally, dark matter is at the core of modern astrophysics. Many well known theoretical models for dark matter exist in the literature, for instance, see [7-29] (the list is by no means exhaustive). Some models that do not hypothesize dark matter appear in [30-38]. Well known density profiles originated in [39-41]. Excellent reviews are to be found in [42-45].

In this paper, we shall revisit the model of perfect fluid dark matter, developed in Ref.[1], in the light of the observed/inferred data of our galaxy. Our analysis would require three ingredients: (i) A method, viz., the Faber-Visser

---

<sup>1</sup>The reason for the appearance of static FLRW background around the imbedded perfect fluid dark matter is already explained in Ref.[1]. The Einstein field equations are solved with perfect fluid stress tensor in both the cases but we sought a static solution from the start. While working on a local problem (flat rotation curve), the scale factor is usually fixed to  $R_0 = 1$  today.

[46] method of combined post-Newtonian measurements of rotation curves and gravitational lensing for measuring the equation of state  $\omega(r)$  of the dark matter and determining the rotation curve mass ( $m_{\text{RC}}$ ), the lens mass ( $m_{\text{Lens}}$ ) and the post-Newtonian mass ( $M_{\text{pN}}$ ). However, for the perfect fluid solution we consider here, it suffices to consider only the rotation curve as a constraint, while the lens mass will be a result of computation. (ii) Two observed inputs, viz., the circular velocity  $\ell$  ( $= 2v_c^2/c_0^2$ ) of probe particles, where  $c_0$  is the speed of light in vacuum, and the radius  $R_{\text{MW}}$  of our galactic halo. (iii) An observational constraint, viz., the one imposed by the attractive nature of dark matter so that  $p/\rho = \omega(r) \geq 0$ . The nature is attractive because the very existence of dark matter is speculated from observations of the Doppler shifted light emanating from neutral hydrogen clouds moving on stable circular orbits in the galactic halo [20,32,47,48]. Using these ingredients, we shall analyze how choices of the adjustable parameter  $D$  appearing in the dark matter metric lead to different types of scenarios.

The following are our new results: Depending on the values of  $D$ , we show that (i) The observable masses  $m_{\text{RC}}$ ,  $m_{\text{Lens}}$  and  $M_{\text{pN}}$  compare well with the masses inferred by other independent means. (ii) There could appear an intriguing negative pressure matter sector ( $\omega < 0$ ) beyond the halo radius<sup>2</sup>. (iii) The Faber-Visser  $\chi$ -factor has values near unity so that pressure contribution to the post-Newtonian mass  $M_{\text{pN}}$  is negligible. Hence the perfect fluid dark matter resembles dust ( $\omega \sim 0$ ) akin to CDM model. (iv) There is flexibility in the halo radius in the sense that our model can accommodate extended radii. All these imply that, fundamentally, the perfect fluid model stands up to actual observations on mass, equation of state and in addition predicts marginal quiescence matter at asymptotic distances, all within a single formulation.

In Sec.2. we briefly outline the perfect fluid dark matter and in Sec.3, display the Faber-Visser post-Newtonian observables. In Sec.4, we apply the galaxy inputs to those observables and deduce the most suitable range of  $D$  that agrees with the observational constraints from our galaxy. In Sec.5, we conclude the paper. We take  $G = 1$ ,  $c_0 = 1$ , unless specifically mentioned.

## 2 Perfect fluid dark matter

The general static spherically symmetric space-time is represented by the following metric

$$ds^2 = -e^{\nu(r)} dt^2 + e^{\lambda(r)} dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad (1)$$

where the functions  $\nu(r)$  and  $\lambda(r)$  are the metric potentials. For the perfect fluid, the matter energy momentum tensor  $T_\beta^\alpha$  is given by  $T_t^t = \rho(r)$ ,  $T_r^r =$

---

<sup>2</sup>However, it will be shown later that the  $\omega < 0$  matter sector is not exotic in nature. It will also be evident that, we can shift the values of  $D$  to make  $\omega < 0$  matter appear at any finite radius beyond halo radius  $R_{\text{MW}}$ , but we must take care not to violate the attractive nature  $\omega \geq 0$  inside  $R_{\text{MW}}$ .

$T_\theta^\theta = T_\varphi^\varphi = p(r)$ , where  $\rho(r)$  is the energy density,  $p(r)$  is the isotropic pressure. Considering flat rotation curve as an input, an exact solution of Einstein field equations is derived in [1]:

$$e^{\nu(r)} = B_0 r^\ell, \quad (2)$$

$$e^{-\lambda(r)} = \frac{c}{a} + \frac{D}{r^a}, \quad (3)$$

$$a = -\frac{4(1+\ell) - \ell^2}{2+\ell}, \quad (4)$$

$$c = -\frac{4}{2+\ell}, \quad (5)$$

$$\ell = 2v_c^2/c_0^2, \quad (6)$$

where  $B_0 > 0$ ,  $D$  are integration constants and  $v_c$  is the circular velocity of stable circular hydrogen gas orbits treated as probe particles. The exact energy density and pressure are

$$\rho(r) = \frac{1}{8\pi} \left[ \frac{\ell(4-\ell)}{4+4\ell-\ell^2} r^{-2} - \frac{D(6-\ell)(1+\ell)}{2+\ell} r^{\frac{\ell(2-\ell)}{2+\ell}} \right] \quad (7)$$

$$p(r) = \frac{1}{8\pi} \left[ \frac{\ell^2}{4+4\ell-\ell^2} r^{-2} + D(1+\ell) r^{\frac{\ell(2-\ell)}{2+\ell}} \right]. \quad (8)$$

The free adjustable parameter  $D$ , having the dimension of  $(\text{length})^{-2}$ , in the solution is extremely sensitive and its value can be decided only by observed physical constraints. In the present case, the constraint is that the galactic fluid be non-exotic and attractive, i.e., the equation of state parameter  $\omega(r) = \frac{p(r)}{\rho(r)} \geq 0$  must hold within the halo radius. With this information at hand, an interesting aspect of the solution can be found from Eqs.(7) and (8).

It can be seen that the integrated quantity, call it  $M_0 = 4\pi \int_0^r \rho(r) r^2 dr$

derived from exact  $\rho(r)$  given by Eq.(7), is identical with the Newtonian mass  $M_N$  derived in Eq.(23) below. One could as well call  $M_N$  the post-Newtonian counterpart of  $M_0$  since  $\rho(r)$  in Eq.(23) is expressed as derivatives of post-Newtonian masses. The question then we ask: What quantity derived from the exact solution differs from its measurable post-Newtonian counterpart? One such quantity is the total mass within a radius  $r$  with *pressure contribution*, which is defined by, using Eqs.(7) and (8)

$$M_{\text{total}}(r) = 4\pi \int_0^r (\rho + 3p) r^2 dr = \frac{\ell(2+\ell)r}{4+4\ell-\ell^2} + \frac{2D}{\ell-6} r^{\frac{4+4\ell-\ell^2}{2+\ell}} \quad (9)$$

$$= \frac{\ell r}{2} + \frac{D\ell r^3}{3} - \frac{\ell^2 r}{4} + O(\ell^2). \quad (10)$$

We shall see in the next section that its post-Newtonian counterpart is just  $M_{\text{pN}}(r) = \frac{\ell r}{2}$ . Hence the theoretical and observable masses in principle differ depending on arbitrary values of  $D$ , even when  $D = 0$ . Therefore, let us proceed to define the post-Newtonian observables.

### 3 Faber-Visser post-Newtonian observables

We shall only quote the relevant expressions here. For details, see Faber-Visser [46]. They considered the metric in the form

$$ds^2 = -e^{2\Phi(r)} dt^2 + \frac{dr^2}{1 - \frac{2m(r)}{r}} + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad (11)$$

which is completely determined by the two metric functions  $\Phi(r)$  and  $m(r)$ . Comparing it with the metric (1), we have

$$m(r) = \frac{r(1 - \frac{c}{a} - \frac{D}{r^a})}{2}, \quad (12)$$

$$\Phi(r) = \frac{\log(B_0) + \ell \log(r)}{2}. \quad (13)$$

The potentials  $\Phi_{\text{RC}}(r)$  and  $\Phi_{\text{Lens}}(r)$ , obtained respectively from the rotation curve data and gravitational lensing observations, are derived to be

$$\Phi_{\text{RC}}(r) = \Phi(r) = \frac{\log(B_0) + \ell \log(r)}{2}, \quad (14)$$

$$\begin{aligned} \Phi_{\text{Lens}}(r) &= \frac{\Phi(r)}{2} + \frac{1}{2} \int \frac{m(r)}{r^2} dr = \frac{\log(B_0) + \ell \log(r)}{4} \\ &+ \frac{D(\ell + 2)r^{\frac{4(1+\ell)-\ell^2}{2+\ell}} + \ell(\ell - 4)\log(r)}{4(\ell^2 - 4\ell - 4)}. \end{aligned} \quad (15)$$

The lensing potential  $\Phi_{\text{Lens}}(r)$  is a fundamental observable quantity. When the pressures and matter fluxes are small compared to the mass-energy density then  $\Phi_{\text{RC}}(r) = \Phi_{\text{Lens}}(r)$ , otherwise they may not be equal.

One pseudo-mass, inferred from rotation curve measurements, is given by

$$m_{\text{RC}}(r) = r^2 \Phi'(r) = \frac{\ell r}{2}. \quad (16)$$

Another pseudo-mass  $m_{\text{Lens}}(r)$ , obtained from lensing measurements, is defined as

$$m_{\text{Lens}}(r) = \frac{r^2 \Phi_{\text{RC}}(r)}{2} + \frac{m(r)}{2} = \frac{r[a(1 + \ell - Dr^{-a}) - c]}{4a}. \quad (17)$$

For the equation of state parameter for perfect fluid, we should evaluate  $\omega$  and impose the constraint that up to  $r = R_{\text{MW}}$ ,

$$\omega(r) = \frac{p_r(r) + 2p_t(r)}{\rho(r)} \geq 0, \quad (18)$$

which will provide a limit on  $D$ . From the first order approximations of Einstein's equations, one obtains [46]

$$\rho(r) = \frac{2m'_{\text{Lens}}(r) - m'_{\text{RC}}(r)}{4\pi r^2} = \frac{r^{(-2-a)} [-cr^a + a(r^a - D) + a^2D]}{8\pi a}, \quad (19)$$

$$\begin{aligned} p_r(r) + 2p_t(r) &= \frac{2[m'_{\text{RC}}(r) - m'_{\text{Lens}}(r)]}{4\pi r^2} \\ &= \frac{r^{(-2-a)} [cr^a - a^2D + a\{(\ell - 1)r^a + D\}]}{8\pi a}. \end{aligned} \quad (20)$$

Then Eq.(18) yields

$$\begin{aligned} \omega(r) &= \frac{p_r(r) + 2p_t(r)}{3\rho(r)} \approx \frac{2}{3} \frac{m'_{\text{RC}}(r) - m'_{\text{Lens}}(r)}{2m'_{\text{Lens}}(r) - m'_{\text{RC}}(r)} \\ &= \frac{cr^a - a^2D + a\{(\ell - 1)r^a + D\}}{3[-cr^a + a(r^a - D) + a^2D]}. \end{aligned} \quad (21)$$

We have intentionally kept in the left hand side of the above Eq.(21) the transverse pressure component  $p_t$  for transparency, remembering that for perfect fluid  $p_r = p_t$ , an exact equality that was used to derive the metric (1).

It is to be emphasized that, *observationally*, such exact equalities as  $p_r = p_t$  are impossible to attain. It follows that the difference in dimensionless pressures is not zero but [46]

$$\begin{aligned} 4\pi r^2 [p_r(r) - p_t(r)] &= \frac{2}{r} (m_{\text{RC}} - m_{\text{Lens}}) - r \left[ \frac{m_{\text{RC}} - m_{\text{Lens}}}{r} \right]' + O\left(\frac{2m}{r}\right)^2 \\ &= \frac{r^{-a} [cr^a + 2a^2D + a\{(\ell - 1)r^a + D\}]}{8\pi a}, \end{aligned} \quad (22)$$

which is just the *post-Newtonian version of isotropicity* of the perfect fluid. However, this value of the right hand side for our galaxy (and presumably for all other samples as well) is exceedingly small but not exactly zero!

The next issue is whether the model is Newtonian or not, that is, how much of pressure contribution to mass is there. For this, we need to compare the two integrals, one is the Newtonian mass  $M_{\text{N}}(r)$  given by, using Eqs.(19) and (20),

$$M_{\text{N}}(r) = 4\pi \int_0^r \rho(r) r^2 dr = \frac{r(a - c - ar^{-a}D)}{2a}, \quad (23)$$

and the other is the mass in the first post-Newtonian approximation [46]

$$M_{\text{pN}}(r) = 4\pi \int_0^r (\rho + p_r + 2p_t) r^2 dr = \frac{\ell r}{2}. \quad (24)$$

Eqs.(14-24) are the needed Faber-Visser post-Newtonian observables to be examined using the available galactic data.

The Faber-Visser  $\chi$ -factor, designed to provide a measure of the size of the pressure contribution, can be obtained from Eq.(21)

$$\chi(r) = \frac{m'_{\text{Lens}}(r)}{m'_{\text{RC}}(r)} = \frac{2 + 3\omega(r)}{2 + 6\omega(r)}. \quad (25)$$

For dust matter, pressures are small so that  $\omega \simeq 0 \Rightarrow \chi(r) \simeq 1$ . Thus, if there is enough pressure in the dark matter,  $\chi(r)$  should have values away from unity.

## 4 Application to our galaxy

Zaritsky [45] collated the published older results (till 1998, see e.g., [49-53]) and demonstrated that they are all consistent with a galactic halo that is nearly isothermal with a characteristic circular velocity oscillating between  $v_c \simeq 180$  to  $220 \text{ km sec}^{-1}$  at 15 kpc. There are however more recent works on constraining the mass and extent of the Milky Way's halo (see e.g., [54,55,56]). We shall use these data in our computations below. Dehnen *et al* [54] suggested a virial radius  $R_{\text{vir}} \sim 200 \text{ kpc}$ , enclosing a virial mass  $M_{\text{vir}} \sim 1.5 \times 10^{12} M_{\odot}$ . We adopt them as the halo radius  $R_{\text{MW}}$  and mass  $M_{\text{MW}}$  of our galaxy.

Xue *et al* [55] observed that the Milky Way's circular velocity curve at  $\sim 60 \text{ kpc}$  gently falls from the adopted value of  $v_c \simeq 220 \text{ km sec}^{-1}$  at the Sun's location to  $v_c \simeq 175 \text{ km sec}^{-1}$  and implies  $M(< 60 \text{ kpc}) = (4.0 \pm 0.7) \times 10^{11} M_{\odot}$ . Deason *et al* [56] infer that the mass within 150 kpc probably lies in the range  $(5 - 10) \times 10^{11} M_{\odot}$ . The measured fall in  $v_c$  is not considered serious because the implied mass ratio between the two extremes is only  $(175/220)^2 = 0.63$ .

Our strategy is to first find  $\omega(r)$  from the Faber-Visser Eq.(21) using the input of  $v_c$  (that is,  $\ell$ ) at some radius  $r$ . Next, within the halo boundary  $R_{\text{MW}} \sim 200 \text{ kpc}$ , we impose the constraint  $\omega(r < 200 \text{ kpc}) > 0$ , which means attractive dark matter halo. At the boundary itself, we impose that  $\omega(R_{\text{MW}}) = 0$ , thereby allowing for a change of sign in  $\omega$  beyond the halo boundary. We then analyze in detail the numerical limits on  $\omega(r)$  using the observed value of  $\ell$  and different signs of the adjustable parameter  $D$ .

Following Xue *et al* [55], we take  $v_c(60 \text{ kpc}) = 175 \text{ km sec}^{-1}$ , which means  $\ell = 2v_c^2/c_0^2 = 6.80 \times 10^{-7}$ . Then from Eq.(21), we get

$$\omega(r) = \frac{6.43 \times 10^{-14} + 0.33Dr^2}{2.26 \times 10^{-7} - Dr^2} > 0, \quad (26)$$

which yields

$$\omega(60 \text{ kpc}) = \frac{1.78 \times 10^{-17} + 0.33D}{6.30 \times 10^{-11} - D} > 0. \quad (27)$$

We now consider three cases of signs of  $D$  and omit mentioning its dimension in what follows.

**Case (1):**  $D = 0$ . This value suggests itself. Then, from Eq.(26), we have  $\omega(r) = 2.8 \times 10^{-7}$  and  $\chi(r) = 1 \forall r$ , which imply that the perfect fluid approximates to dust dark matter. Because of negligible pressure, as evidenced by the Faber-Visser function  $\chi(r) = 1$ , this case is more consistent with the Cold Dark Matter (CDM) paradigm for galactic fluid [57]. We now use  $\ell = 6.80 \times 10^{-7}$ ,  $D = 0$  in the expressions for the masses  $m_{\text{RC}}$ ,  $m_{\text{Lens}}$ ,  $M_{\text{pN}}(r)$  and find that all have nearly the same values<sup>3</sup> within  $r = 60$  kpc, viz.,  $m_{\text{RC}} = m_{\text{Lens}} = M_{\text{N}} = M_{\text{pN}} = 4.25 \times 10^{11} M_{\odot}$  (Fig.1). The last two equalities suggest that the model is Newtonian, that is, pressure contribution is negligible [see Eqs.(23,24)]. Within the current level of uncertainties in the values of observed mass, it is evident that our common value is quite comparable with the value  $M (< 60 \text{ kpc}) = (4.0 \pm 0.7) \times 10^{11} M_{\odot}$  inferred by Xue *et al* [55]. Assuming no further significant fall-off in the circular velocity from  $v_c \simeq 175 \text{ km sec}^{-1}$  (distinct from the radial velocity dispersion via Jean's law), we find that at  $r = 150 \text{ kpc}$ ,  $m_{\text{RC}} = m_{\text{Lens}} = M_{\text{pN}} = 1.0 \times 10^{12} M_{\odot}$ . This mass value is reasonably consistent with the range  $(5 - 10) \times 10^{11} M_{\odot}$  suggested by Deason *et al* [56].

Using  $R_{\text{MW}} \sim 200 \text{ kpc}$  [54], and  $\ell = 6.80 \times 10^{-7}$  [55],  $D = 0$ , we find that,  $m_{\text{RC}} = m_{\text{Lens}} = M_{\text{pN}} \sim 1.4 \times 10^{12} M_{\odot}$  enclosed within the radius  $R_{\text{MW}}$  (Fig.1). This mass value compares well with the virial mass  $M_{\text{vir}} \sim 1.5 \times 10^{12} M_{\odot}$  obtained by Dehnen *et al* [54],  $M_{\text{vir}} = 1.0^{+0.3}_{-0.2} \times 10^{12} M_{\odot}$  obtained by Xue *et al* [55], which is also comparable to the value  $M_{\text{vir}} = (1.26 \pm 0.24) \times 10^{12} M_{\odot}$  obtained by McMillan [58] using a Bayesian approach. Next, as is evident from Eqs.(10) and (24), generically,  $M_{\text{total}}(r) \neq M_{\text{pN}}(r)$  and it holds even in the case  $D = 0$ , though the difference is negligible. Also, from Eq.(22), we find  $4\pi r^2 [p_r(r) - p_t(r)] \sim 10^{-14} \forall r$ , an exceedingly small value consistent with the assumption of pressure isotropy (Fig.2). However, nothing peculiar happens in  $\omega$  at and beyond the halo boundary  $R_{\text{MW}} \sim 200 \text{ kpc}$ , because  $\omega = 2.8 \times 10^{-7} \forall r$ .

Another range suggested by Eq.(27) is  $0 < D < 6.30 \times 10^{-11}$ , which also leads to  $\omega(r) > 0$  for  $r < 60 \text{ kpc}$  from Eq.(26) but then there will be an unphysical singularity in  $\omega(r)$  appearing at  $r_{\text{sing}} = \sqrt{2.26 \times 10^{-7}/D}$ , hence discarded here.

**Case (2):**  $D < 0$ . We impose  $\omega(R_{\text{MW}}) = 0$  ending the extent of dark matter at the halo radius  $R_{\text{MW}} \sim 200 \text{ kpc}$ . This boundary condition yields, from Eq.(26),

$$\omega(200 \text{ kpc}) = \frac{1.6 \times 10^{-18} + 0.33D}{5.67 \times 10^{-12} - D} = 0, \quad (28)$$

leading to a fixed value  $D = -4.84 \times 10^{-18}$ . This value, when put back in Eq.(26), leads to three different sectors:  $\omega(r) > 0$  for  $r \in [0, 200 \text{ kpc})$ ,  $\omega(200$

---

<sup>3</sup>Note that the mass formulas in this paper are given in terms of distance  $r$  kpc in relativistic units, but it is preferable to use the conventional and direct unit of solar mass  $M_{\odot}$ . Therefore, we use the conversion  $1 \text{ kpc} = 2.084 \times 10^{16} M_{\odot}$  and re-express the masses in terms of  $M_{\odot}$  in Sec.4.



kpc) = 0 and  $\omega(r > 200 \text{ kpc}) < 0$  (Fig.3). This case has several interesting features and Fig.3 is the most eloquent illustration of how the constraint Eq.(28) can eventually determine the behavior of matter in all distance sectors.

First, the sector having  $\omega(r) < 0$  has positive energy density  $\rho(r) > 0$  and negative pressure  $p_r(r) + 2p_t(r) < 0$  (Fig.4), but the matter is *not* exotic as it still does not violate the Null Energy Condition (NEC)<sup>4</sup>(Fig.5). Interestingly, the matter does not resemble either the cosmological phantom ( $\omega < -1$ ) or quintessence matter ( $\omega < -1/3$ ) because, as is evident from Fig.3,  $\omega(r) \sim -10^{-7}$  at any finite  $r > 200 \text{ kpc}$ . However, at  $r \rightarrow +\infty$ , we find from Eq.(26) that  $\omega(\infty) \rightarrow (-1/3)+$ , which therefore *marginally* corresponds to quintessence matter, and there appears no singularity in  $\omega(r)$  at any radius. Second, Fig.6 shows  $\chi(r) \simeq 1$  for  $r \in [0, 400 \text{ kpc}]$ , indicating that the pressure contribution is quite insignificant, thereby once again supporting the CDM paradigm. Third, it can be easily seen from Eq.(24) that the measure  $M_{\text{pN}}(r)$  gives a comparable galactic mass  $\sim 1.4 \times 10^{12} M_\odot$  enclosed within the radius  $R_{\text{MW}}$ , while other mass estimates are also very close to it. Fourth, from Eq.(22), it follows that  $4\pi r^2 [p_r(r) - p_t(r)] \sim 10^{-14}$ , a very minute difference as expected of perfect fluid from the observational point of view. Finally, note that the value of  $D$  actually determines the terminating radius where  $\omega(r) = 0$ . For example, for  $D > -4.84 \times 10^{-18}$ , the radius can be arbitrarily shifted away from  $R_{\text{MW}} \sim 200 \text{ kpc}$  (for an illustration, see Fig.7). This means that  $D$  can be adjusted to the possibility of having a larger Milky Way halo than considered here (viz., 200 kpc).

In view of these merits, we can say that the range  $-4.84 \times 10^{-18} \leq D \leq 0$ , which in turn leads to  $0 \leq \omega(r) \leq 2.8 \times 10^{-7}$  for the perfect fluid dark matter, can reasonably describe our galactic halo. The simple physical requirement of an attractive halo thus leads to a strong constraint indicating a dust-like dark matter ( $\omega \sim 0$  or  $p \sim 0$ ).

We wish to point out here that so far we focused only on the constraint from the Milky Way, but there must be constraints from, for example, the Cosmic Microwave Background (CMB) on deviations from  $\omega = 0$  for dark matter. Using CMB, supernovae Ia and large scale structure data in a fluid dark matter model, Müller [59] found constraints on  $\omega$  as follows:  $-1.50 \times 10^{-6} < \omega < 1.13 \times 10^{-6}$  if the dark matter produces no entropy and  $-8.78 \times 10^{-3} < \omega < 1.86 \times 10^{-3}$  if the adiabatic sound speed vanishes, both at  $3\sigma$  confidence level. Clearly, we see that both the ranges in [58] concentrate around the value  $\omega \sim 0$ , which is in very good agreement with our result of a dust-like halo. By observing effects of perturbations on CMB and matter power spectra, Kumar and Lu [60] conclude that the current observational data favor the CDM scenario with the cosmological constant type dark energy at the present epoch. This is the most recent result on the CMB constraint.

**Case (3):**  $D > 0$ . Eq.(28) then suggests  $0 < D < 5.67 \times 10^{-12}$  giving  $\omega(200 \text{ kpc}) > 0$ . Let us take a concrete value, for example,  $D = 5 \times 10^{-12}$  to see what

---

<sup>4</sup>NEC is defined by  $\rho + p_r \geq 0$  and  $\rho + p_t \geq 0$ . Matter that violates these conditions is called "exotic".

that means. We find from Eq.(26) that  $\omega(200 \text{ kpc}) > 0$  but  $\omega \rightarrow \infty$  occurs at  $r_{\text{sing}} = 212 \text{ kpc}$ . Fig.8 shows the occurrence of cosmological quintessence matter ( $\omega < -1/3$ ) immediately beyond 212 kpc. If  $D > 5 \times 10^{-12}$ , then  $\omega(r) < 0$  inside the halo boundary, both contrary to our assumption that  $\omega(200 \text{ kpc}) = 0$ . Fig.9 then shows that  $M_{\text{pN}}(r)$  is larger than  $M_{\text{N}}(r)$ , meaning that there is *substantial pressure* in the halo, as confirmed also by the  $\chi$ -factor that shows values away from unity, signalling the presence of non-negligible pressure as opposed to the CDM paradigm. All these features could make the case truly intriguing if one is ready to live with a singularity in  $\omega(r)$ . One might consider any other allowed value respecting  $D < 5.67 \times 10^{-12}$ , say  $D = 2.83 \times 10^{-12}$  (This particular value corresponds to  $\omega(200 \text{ kpc}) = \frac{1}{3}$ ), then we find from Eq.(26) that the singularity just shifts to a larger radius  $r > 200 \text{ kpc}$ , and quintessence matter begins to appear from that radius onwards, as shown in Fig.10. However, as we see the unphysical singularity can only be arbitrarily shifted at will by choosing  $D$  but not removed.

## 5 Conclusions

We revisited the perfect fluid dark matter model in the light of the Faber-Visser post-Newtonian formalism that requires simultaneous measurement of pseudo-mass profiles from rotation curve and gravitational lensing observations. However, for the model considered here, no constraints from lensing were used. The lensing mass together with other masses were computed using recent data. The formalism provides information of the equation of state of the galactic fluid, especially the pressure component in it. We saw above how the variation of a crucial parameter  $D$ , that has dimension of the cosmological constant  $(\text{kpc})^{-2}$ , can lead to different scenarios. We deduced the post-Newtonian version of isotropicity in Eq.(22) and computed the equation of state parameter  $\omega(r)$ , the observables such as the post-Newtonian mass  $M_{\text{pN}}$ , the rotation curve and lens pseudo-masses from Eqs.(21,24,16,17) respectively in terms of the metric functions<sup>5</sup>.

The computation of the above observables for our galaxy was done taking into account the data on rotational velocity  $\ell = 6.80 \times 10^{-7}$  from Xue *et al* [55] and requiring an attractive halo ( $\omega(r) \geq 0$ ) at least within the halo radius  $R_{\text{MW}} \sim 200 \text{ kpc}$  [54]. The choice of the values of  $D$  obtained from Eqs.(27,28), when used in Eq.(26), led to the profiles of  $\omega(r)$ . Salient features of our analysis are summarized below:

The case  $D = 0$  in Eq.(27) immediately led to  $\omega(r) = 2.8 \times 10^{-7}$  and  $\chi(r) = 1 \forall r$ , which imply that the perfect fluid approximates to dust dark matter. The masses within  $r = 60 \text{ kpc}$ , viz.,  $m_{\text{RC}} = m_{\text{Lens}} = M_{\text{N}} = M_{\text{pN}} = 4.25 \times 10^{11} M_{\odot}$  are found to be in excellent agreement with the value  $M (< 60$

---

<sup>5</sup>While the inferred pseudo-masses pertain to the same galaxy, they refer to different radii, hence incomparable. The situation is likely to improve in the future when observations with a higher resolution will be carried out (see for details, [46]).

kpc) =  $(4.0 \pm 0.7) \times 10^{11} M_{\odot}$  inferred by Xue *et al* [55]. The mass within  $r = 150$  kpc obtained by Deason *et al* [55] as well as the virial mass  $M_{\text{vir}}$  from Dehnen *et al* [54] are also found to be quite comparable (Fig.1).

The case  $D < 0$  has a number of implications. The value  $D = -4.84 \times 10^{-18}$  corresponds to  $\omega(200 \text{ kpc}) = 0$  ending dark matter halo. Eq.(26) then leads to three different sectors:  $\omega(r) > 0$  for  $r \in [0, 200 \text{ kpc})$ ,  $\omega(200 \text{ kpc}) = 0$  and  $\omega(r > 200 \text{ kpc}) < 0$  (Fig.3). The last sector has positive energy density  $\rho(r) > 0$  and negative pressure  $p_r(r) + 2p_t(r) < 0$  (Fig.4), but the matter is *not* exotic as it still does not violate the Null Energy Condition (NEC)(Fig.5). For  $D > -4.84 \times 10^{-18}$ , the halo radius can be arbitrarily shifted away from 200 kpc (Fig.7), which means that  $D$  can be adjusted to the possibility of having a larger Milky Way halo than considered here.

The case  $D > 0$  signals the presence of non-negligible pressure in the halo as opposed to the CDM paradigm but also leads to a singularity in  $\omega(r)$  that can only be arbitrarily shifted at will by choosing  $D$  but not removed.

In view of the consistency with the recent galactic data and flexibility as above, we suggest an overall range  $-4.84 \times 10^{-18} \leq D \leq 0$ , which in turn leads to  $0 \leq \omega(r) \leq 2.8 \times 10^{-7}$  for the perfect fluid singularity-free equation of state of dark matter. As we see, the values are concentrated around  $D \sim 0$  leading to a strong constraint of dust-like dark matter ( $\omega \sim 0$ ), which is supported also by CMB constraints [59,60]. This is the main result of our paper.

We recall that an acceptable practical, working definition of a galactic halo is still debatable [45]. One theoretically sound definition is that the halo is the volume enclosing all of the mass that has already decoupled from the Hubble flow. Another definition is the virial radius enclosing gravitationally bound halo mass. All of them are problematic for practical measurements. The only viable solution is to altogether avoid defining the halo as a discrete entity. Instead, one should focus on the mass profile or on the mass within a selected, fixed radius. However, all these arguments do not rule out a future observation of a terminated discrete halo, even though it might extend to hundreds of kpc farther than the adopted  $R_{\text{MW}} = 200 \text{ kpc}$ . Within this ideology, the interval  $0 \leq \omega(r) \leq 2.8 \times 10^{-7}$  allowing flexibility in the halo radius does make good sense.

## 6 Acknowledgment

The authors wish to thank an anonymous referee for his/her insightful comments that led to significant improvements over the initial version.

## 7 References

- [1] F. Rahaman, K. K. Nandi, A. Bhadra, M. Kalam and K. Chakraborty, Phys. Lett. B **694**, 10 (2010).
- [2] T. Harko and F.S. N. Lobo, Phys. Rev. D **83**,124051 (2011).

- [3] A.X. González-Morales and D. Nuñez, J. Phys. Conf. Ser. **229**, 012041(2010).
- [4] A. Avelino, Y. Leyva and L. A. Ureña-López, Phys. Rev. D **88**, 123004 (2013).
- [5] J. Oort, Bull. Astron. Inst. Netherl. **6**, 155 (1931).
- [6] F. Zwicky, Astrophys. J. **86**, 217 (1937).
- [7] T. Matos and F. S. Guzmán, Annalen d. Phys. **9**, S1 (2000).
- [8] F.S. Guzmán and L.A. Ureña-López, Phys. Rev. D **68**, 024023 (2003).
- [9] T. Harko, F.S.N. Lobo, M.K. Mak and S.V. Sushkov, Mod. Phys. Lett. A **29**, 1450049 (2014).
- [10] R. Izmailov, A.A. Potapov, A.I. Filippov, M. Ghosh and K.K. Nandi, Mod. Phys. Lett. A **30**,1550056 (2015).
- [11] A.A. Potapov, R. Izmailov, O. Mikolaychuk, N. Mikolaychuk, M. Ghosh and K.K. Nandi, JCAP 07(2015) 018.
- [12] S. Nojiri and S.D. Odintsov, Phys. Rept. **505**, 59 (2011).
- [13] R.B. Metcalf and J. Silk, Phys. Rev. Lett. **98**, 071302 (2007).
- [14] S. Bharadwaj and S. Kar, Phys. Rev. D **68**, 023516 (2003).
- [15] M. Colpi, S.L. Shapiro and I. Wasserman, Phys. Rev. Lett. **57**, 2485 (1986).
- [16] G. Efstathiou, W.J. Sutherland and S.J. Maddox, Nature (London) **348**, 705 (1990).
- [17] T. Matos, F.S. Guzmán and D. Nuñez, Phys. Rev. D **62**, 061301 (2000); T. Matos and L. A. Ureña-Lopez, Phys. Lett. B **538**, 246 (2002).
- [18] P.J.E. Peebles, Phys. Rev. D **62**, 023502 (2000).
- [19] U. Nucamendi, M. Salgado and D. Sudarsky, Phys. Rev. D **63**,125016 (2001).
- [20] E.W. Mielke and F.E. Schunck, Phys. Rev. D **66**, 023503 (2002).
- [21] J.E. Lidsey, T. Matos and L.A. Ureña-Lopez, Phys. Rev. D **66**, 023514 (2002).
- [22] A. Arbey, J. Lesgourgues and P. Salati, Phys. Rev. D **68**, 023511 (2003).
- [23] M.K. Mak and T. Harko, Phys. Rev. D **70**, 024010 (2004).
- [24] G. Dvali, G. Gabadadze and M. Porrati. Phys. Lett. B **484**, 112 (2000); N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B **429**, 263 (1998); I. Antoniadis, Phys. Lett. B **436**, 257 (1998); I. Antoniadis, S. Dimopoulos and G. Dvali. Nucl. Phys. B **516**, 70 (1998).
- [25] K.K. Nandi, A.I. Filippov, F. Rahaman, S. Ray, A.A. Usmani *et al.*, Mon. Not. Roy. Astron. Soc. **399**, 2079 (2009).
- [26] J.-c. Hwang and H. Noh, Phys. Lett. B **680**, 1 (2009).
- [27] S.L. Dubovsky, P.G. Tinyakov and I.I. Tkachev, Phys. Rev. Lett. **94**,181102 (2005).
- [28] S. Dodelson and L.M. Widrow, Phys. Rev. Lett. **72**,17 (1994).
- [29] S. Dodelson, G. Gyuk and M.S. Turner, Phys. Rev. Lett. **72**, 3754 (1994).
- [30] P.D. Mannheim, Prog. Part. Nucl. Phys. **56**, 340 (2006).
- [31] P.D. Mannheim and J.G. O'Brien, Phys. Rev. Lett. **106**, 121101 (2011).
- [32] K.K. Nandi and A. Bhadra, Phys. Rev. Lett. **109**, 079001 (2012).

- [33] M. Milgrom, *Astrophys. J.* **270**, 365 (1983); *ibid.* **270**, 371 (1983); *ibid.* **270**, 384 (1983).
- [34] R.H. Sanders, *Astrophys. J.* **473**, 177 (1996).
- [35] R.A. Swaters, R.H. Sanders and S.S. McGaugh, *Astrophys. J.* **718**, 380 (2010).
- [36] J.W. Moffat, *JCAP* 03 (2006) 004.
- [37] J.R. Brownstein and J.W. Moffat, *Astrophys. J.* **636**, 721 (2006).
- [38] S. Capozziello, V.F. Cardone and A. Troisi, *Mon. Not. Roy. Astron. Soc.* **375**, 1423 (2007).
- [39] A. Burkert, *Astrophys. J.* **447**, L25 (1995).
- [40] P. Salucci and A. Burkert, *Astrophys. J.* **537**, L9 (2000).
- [41] J.F. Navarro, C.S. Frenk and S.D.M. White, *Astrophys. J.* **490**, 49 (1997).
- [42] G. Jungman, M. Kamionkowski and K. Griest, *Phys. Rept.* **267**, 195 (1996).
- [43] L.E. Strigari, *Phys. Rept.* **531**, 1 (2013)
- [44] G. Bertone, D. Hooper and J. Silk, *Phys. Rept.* **405**, 279 (2005).
- [45] D. Zaritsky, *Invited Review for The Third Stromlo Symposium: The Galactic Halo* (1998) [arXiv:astro-ph/9810069].
- [46] T. Faber and M. Visser, *Mon. Not. Roy. Astron. Soc.* **372**, 136 (2006).
- [47] K. Lake, *Phys. Rev. Lett.* **92**, 051101 (2004).
- [48] T. Faber [arXiv:gr-qc/0607029].
- [49] J. Einasto and D. Lynden-Bell, *Mon. Not. Roy. Astron. Soc.* **199**, 67 (1982).
- [50] S. Raychaudhury and D. Lynden-Bell, *Mon. Not. Roy. Astron. Soc.* **240**, 195 (1989).
- [51] P.J.E. Peebles, *Astrophys. J.* **449**, 52 (1995).
- [52] E.J. Shaya, P.J.E. Peebles and R.B. Tully, *Astrophys. J.* **454**, 15 (1995).
- [53] M. Fich and S. Tremaine, *Ann. Rev. Astron. Astrophys.* **29**, 409 (1991).
- [54] W. Dehnen, D.E. McLaughlin and J. Sachania, *Mon. Not. Roy. Astron. Soc.* **369**, 1688 (2006).
- [55] X.-X. Xue *et al*, *Astrophys. J.* **684**, 1143 (2008).
- [56] A.J. Deason *et al*, *Mon. Not. Roy. Astron. Soc.* **425**, 2840 (2012).
- [57] S. Dodelson, E. Gates and M.S. Turner, *Science* **274**, 69 (1996).
- [58] P.J. McMillan, *Mon. Not. Roy. Astron. Soc.* **414**, 2446 (2011).
- [59] C.M. Müller, *Phys. Rev. D* **71**, 047302 (2005).
- [60] S. Kumar and L. Xu, *Phys. Lett. B* **737**, 244 (2014).